Available online at www.sciencedirect.com

SCIENCE DIRECT

International Journal of Pressure Vessels and Piping 82 (2005) 379–385

INTERNATIONAL JOURNAL OF Pressure Vessels and Piping

www.elsevier.com/locate/ijpvp

Wall thickness optimization of thick-walled spherical vessel using thermo-elasto-plastic concept

M.H. Kargarnovin*, A. Rezai Zarei, H. Darijani

Department of Mechanical Engineering, Sharif University of Technology, Center of Excellence in Design, Robotic and Automaton, Azadi Ave. P.O. Box 11365-9567, Tehran 11365, I.R. Iran

Received 22 January 2004; revised 6 July 2004; accepted 11 October 2004

Abstract

A study of thick-walled spherical vessels under steady-state radial temperature gradients using elasto-plastic analysis is reported. By considering a maximum plastic radius and using the *thermal autofrettage* method for the strengthening mechanism, the optimum wall thickness of the vessel for a given temperature gradient across the vessel is obtained. Finally, in the case of thermal loading on a vessel, the effect of convective heat transfer on the optimum thickness is considered, and a general formula for the optimum thickness and design graphs for several different cases are presented.

Keywords: Heat transfer; Elasto-plastic; Thick-walled vessels; Temperature gradient

1. Introduction

In general, the existence of any temperature gradient across the wall of a thick-walled vessel induces a thermal stress. Often, thermal stresses are greater than those generated by application of either internal and/or external pressure. From an economical point of view, the thermo-elasto-plastic method is used for design of such vessels. Detailed analyses of thermal stress in spherical and cylindrical vessels in the elastic range are given in [1–4]. In [5] the behaviour of thick-walled spherical and cylindrical vessels under thermal and mechanical stresses is considered. The exact solution for the stress distribution in a thick-walled sphere made of elastic-perfectly plastic material and under a steady state, radial temperature gradient is obtained in [6]. In the same paper an approximate solution with negligible elastic strain is also examined; the approximate and exact solutions yield the same results as the temperature gradient approaches infinity. The onset of yield in thick-walled spherical vessels for various combinations of temperature and pressure and various radius ratios is studied in [7]. Elasto-plastic thermal stresses in a spherical vessel under a temperature gradient across the wall thickness are studied in [8]. In all of these studies, for the thermal stress analysis, the temperature of the external surface of the vessel is held constant, at the same time, the temperature of its internal surface is increased and the resultant stresses are obtained [6–8]. In this paper, after reviewing some reported works in this field, by considering a maximum plastic radius and using the concept of thermal autofrettage for the strengthening mechanism, the analysis of thick-walled spheres with no convective heat transfer to the ambient is presented. Modeling and closed form solutions for stress distributions in the elastic part, due to combined pressure and temperature gradient, thermal loading and unloading and design curves are covered in Sections 2.1–2.5. However, in practice convective heat transfer occurs between the external surface of the vessel and the surroundings. This means that any changes in the temperature of the internal surface will change the temperature of the external surface. This change, in turn, is a function of the internal temperature, the size of the external surface, the mechanical properties of the vessel's material, and the properties of the fluid, which is in contact with the external surface of the vessel. In Section 3,

* Corresponding author. Tel.: +98 21 616 5510; fax: +98 21 600 0021.
E-mail address: mhkargar@sharif.edu (M.H. Kargarnovin).

0308-0161/$ - see front matter © 2004 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijpvp.2004.10.001

the effect of convective heat transfer on the minimum thickness of the vessel, and design graphs for several different cases are presented.

2. Governing stress–strain relations

The following assumptions are made:

(1) The loading and geometry are symmetric, i.e. r and θ in spherical coordinates are principal planes and directions.
(2) Body forces are negligible.
(3) The vessel deformation is quasi-static, i.e. no stress waves are produced by applying a temperature gradient.
(4) The temperature of the inside surface of the vessel is greater than that of the outside surface, i.e. the direction of heat flow is outward.
(5) The temperature in the vessel is such that the effects of creep deformation are not considered.
(6) The vessel material is elastic-perfectly plastic.
(7) The yield stresses in tension and compression are the same, i.e. the Baushinger effect is neglected.
(8) The effects of stress and temperature on the modulus of elasticity, yield stress, coefficient of thermal expansion and the other material properties are negligible.
(9) The Tresca and von-Mises yield criteria are used.

2.1. The onset of yield under the coupled effect of a temperature Gradient and internal pressure

When a thick-walled sphere is under a severe temperature gradient, elastic and plastic zones are established. At the elasto-plastic interface, a radial stress is created that has a similar effect to an internal pressure on the elastic part of the vessel. In addition, because of the difference between the temperatures of the elasto-plastic interface and the outside radius, the elastic part of the vessel is under a temperature gradient. Therefore, the elastic part of the vessel is under the coupled effects of a temperature gradient and an internal pressure. In [7] this subject is studied in detail and the conditions for the onset of yield for different combinations of temperature and pressure are examined. The elastic stresses for combined loading of a thick-walled vessel are obtained from the following relations [7]:

$$\sigma_r/\beta = [m^3(1-p/\beta) - mR^2(m^2+m+1) + R^3(p/\beta + m^2 + m)]/R^3(m^3-1) \tag{1}$$

$$\sigma_\theta/\beta = [-m^3(1-p/\beta) - mR^2(m^2+m+1) + 2R^3(p/\beta + m^2 + m)]/2R^3(m^3-1) \tag{2}$$

In these relations $R=r/a$, $m=b/a$, and $\beta=\alpha E\Delta T/(1-\nu)$; a, b, ΔT, p, and α are the inside radius, outside radius, temperature difference between inside and outside radii, internal pressure, and heat expansion coefficient of the material, respectively. Denoting τ_r as the maximum shear stress at radius r, we have [7]:

$$\tau_r/\beta = (\sigma_\theta - \sigma_r)/2\beta = [3m^3(p/\beta - 1) + mR^2(m^2+m+1)]/4R^3(m^3-1) \tag{3}$$

First yield occurs at the radius at which τ_r reaches the value corresponding to the Tresca or von Mises yield criterion. For different combination of p and β and the radius ratio m, this radius can be at any position within the vessel wall. Fig. 1 shows these critical radii [7]. In regions I and IV, first yield is at the inside radius of the vessel. In region III, first

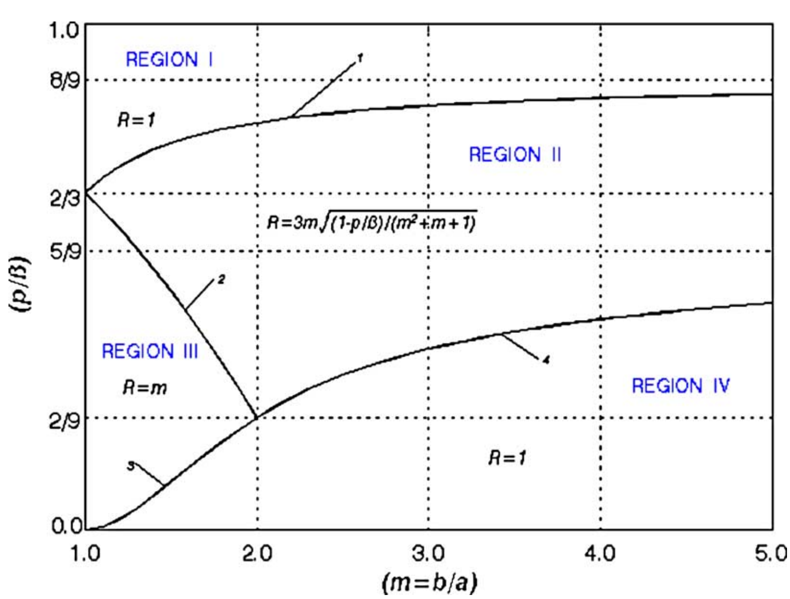

Fig. 1. Different regions for the onset of yield in a thick-walled sphere under combined internal pressure and radial temperature gradient.

yield is at the outside radius. In region II, first yield is at the radius $R=3m\sqrt{(1-p/\beta)/(m^2+m+1)}$. Curves 1–4 are the interfaces of these regions and are defined by [7]:

$$1\to p/\beta=1-(m^2+m+1)/9m^2 \quad (4)$$

$$2\to p/\beta=1-(m^2+m+1)/9 \quad (5)$$

$$3\to p/\beta=2(m-1)^2/3(m^2-m+1) \quad (6)$$

$$4\to p/\beta=(5-4/m-4/m^2)/9 \quad (7)$$

2.2. Elastic thermal stress distribution

The elastic stresses in a thick-walled sphere under thermal loading, ΔT, are [8]:

$$\sigma_r=\beta ab[a+b-(a^2+ab+b^2)/r+a^2b^2/r^3]/(b^3-a^3) \quad (8)$$

$$\sigma_\theta=\beta ab[a+b-(a^2+ab+b^2)/2r-a^2b^2/2r^3]/(b^3-a^3) \quad (9)$$

The temperature distribution in the wall of the vessel is [8]:

$$T(r)=\frac{b/r-1}{b/a-1}\Delta T+T_b \quad (10)$$

2.3. Elasto-plastic thermal stresses

If c is the radius to the interface of the elasto-plastic regions and y is the initial yield stress, the yield criterion and equilibrium equation are satisfied in the plastic part of the vessel ($a\le r\le c$). By combining these relations

$$\mathrm{d}\sigma_r=-2y\,\mathrm{d}r/r \quad (11)$$

and by applying the following boundary condition:

$$\sigma_r=0 \text{ at } r=a \quad (12)$$

The plastic stresses are [8]:

$$\sigma_r=-2y\ln(r/a) \quad (13)$$

$$\sigma_\theta=-y(1+2\ln(r/a)) \quad (14)$$

To obtain the elastic stresses (in an elastic sphere with internal radius c, external radius b, $m_1=b/c$, and $R_1=r/c$), the temperature between radii b and c, and the radial stress at the elasto-plastic interface, c, should be calculated. These unknowns can be obtained from Eqs. (10) and (13), respectively [8]:

$$\beta_c=\beta(m_1-1)/(m-1) \quad (15)$$

$$p_c=2y\ln(c/a)=2y\ln(m/m_1) \quad (16)$$

where $\beta_c=\alpha E(T_c-T_b)/(1-\nu)$, and p_c is the internal pressure for the elastic part of the vessel. Finally, by employing Eqs. (1) and (2), the elastic stresses within the region ($c\le r\le b$), are obtained as [8]:

$$\sigma_r/\beta_c=[m_1^3(1-p_c/\beta_c)-m_1R_1^2(m_1^2+m_1+1)+R_1^3(p_c/\beta_c+m_1^2+m_1)]/R_1^3(m_1^3-1) \quad (17)$$

$$\sigma_\theta/\beta_c=[-m_1^3(1-p_c/\beta_c)-m_1R_1^2(m_1^2+m_1+1)+2R_1^3(p_c/\beta_c+m_1^2+m_1)]/2R_1^3(m_1^3-1) \quad (18)$$

By applying the Tresca yield condition at $r=c$ one can get [8]:

$$-y/\beta_c=[3m_1^3(p_c/\beta_c-1)+m_1(m_1^2+m_1+1)]/2(m_1^3-1) \quad (19)$$

Finally, the variation of internal pressure to thermal loading ratio, i.e. p_c/β_c versus m_1 can be obtained by combining Eqs. (15), (16) and (19) [8]:

$$p_c/\beta_c=[m_1(2m_1^2-m_1-1)\ln(m/m_1)]/\times[m_1^3(3\ln(m/m_1)+1)-1] \quad (20)$$

In Fig. 2 for several different radius ratio m (2, 2.791, 4), the variation of p_c/β_c versus m_1 is plotted.

If the temperature of the inner surface of the vessel is increased such that in the elastic part of the vessel another yield surface is initiated, at inception of this condition we consider the value of c as c^{**} which is the maximum plastic radius and consequently we take m_1 as $m_1^{**}=b/c^{**}$. The value of c^{**} can be obtained by simultaneous solution of Eqs. (6) or (7) and (20). As shown in Fig. 2 for $m<2.791$, m_1^{**} is obtained from Eqs. (6) and (20). On the other hand, if $m>2.791$, m_1^{**} is obtained by solving Eqs. (7) and (20). Note that, Eqs. (13)–(16) are valid for radius $c<c^{**}$ and to determine the value of c, Eq. (16) is divided by Eq. (15) and the result is equated to Eq. (20). By solving this equation, c can be determined and after inserting this in Eq. (16), p_c can be obtained.

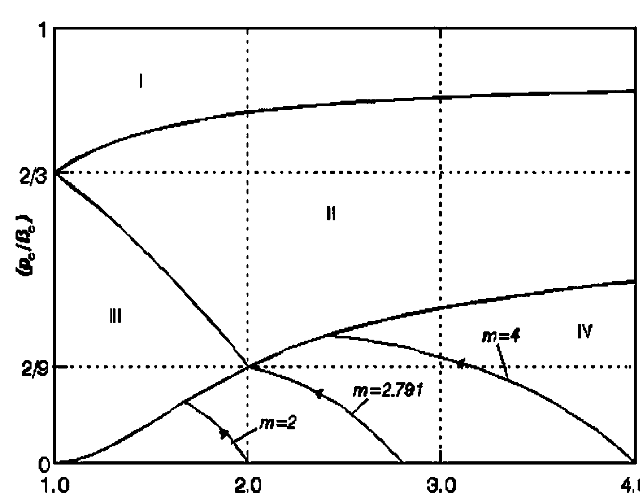

Fig. 2. Variation of p_c/β_c versus m_1.

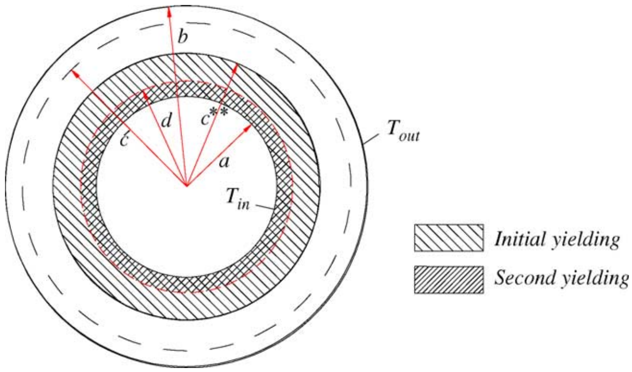

Fig. 3. Different parameters in a thick-walled sphere under temperature gradient.

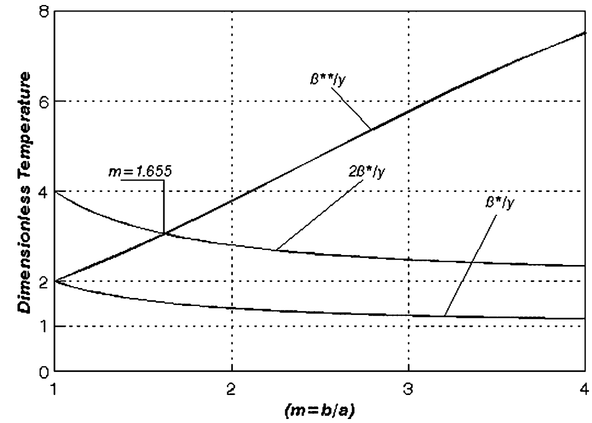

Fig. 4. Variation of dimensionless temperature gradient versus m [9].

2.4. Establishing design curves

Fig. 3 shows a section of a thick-walled spherical vessel and its geometrical parameters in a more general form. The inner and outer radii are a and b, respectively. T_{in} and T_{out} are the inside and outside surface temperatures of the vessel, respectively. As mentioned before, the radius c^{**} represents the primary elasto-plastic interface radius at which a new secondary elasto-plastic region at radius $c^{**}<c'<b$ in the elastic zone of the vessel is created. The radius d is a new generated elasto-plastic interface during unloading of the vessel.

Usually in such vessels, a primary loading and unloading cycle called a pre-working cycle is utilized as a strengthening mechanism. On the other hand, to create an appropriate residual stress across the wall of the vessel, the method of thermal autofrettage is used. In practice, the designer intends to consume less material in the production of the vessel. In this case, for an arbitrary value of dimensionless temperature gradient β, we decrease the wall thickness of the vessel such that a new elasto-plastic interface at $r=c'$ in the elastic part of the vessel is formed. Under this condition, β will be designated by β^{**}. In order to obtain β^{**}, we first substitute Eq. (16) into Eq. (19), then we have:

$$\beta_c = 2y[m_1^3(3\ln(m/m_1)+1)-1]/[m_1(2m_1^2-m_1-1)] \tag{21}$$

Then by combining Eqs. (15) and (21) β becomes:

$$\beta = \frac{m-1}{m_1-1}\,\frac{2y[m_1^3(3\ln(m/m_1)+1)-1]}{[m_1(2m_1^2-m_1-1)]} \tag{22}$$

Therefore, from a design point of view we have [9]:

$$\beta^{**} = \frac{m-1}{m_1^{**}-1}\,\frac{2y[m_1^{**3}(3\ln(m/m_1^{**})+1)-1]}{[m_1^{**}(2m_1^{**2}-m_1^{**}-1)]} \tag{23}$$

If the necessary temperature gradient for initial yielding of the vessel with its material behaving elastic-perfectly plastic is β^*, the necessary condition to have secondary yielding during unloading is to remove a temperature gradient which is greater than $2\beta^*$. Using the yield condition at the inside radius a, the value of β^* is obtained as [9]:

$$\beta^* = 2y(m^2+m+1)/(2m^2+m) \tag{24}$$

Fig. 4 shows the variation of β^*/y, $2\beta^*/y$, and β^{**}/y versus m.

A close study of Fig. 4 reveals that;

- The curve β^*/y shows the variation of temperature gradient versus m for the case of initial yielding of the vessel. It is expected that by increasing wall thickness, the initial thermal load increases but as seen from this figure this is not always true. In addition, the maximum value of β/y for the case of elastic design of the vessel is 2, and hence for values of β/y which are greater than 2, we cannot use the elastic method in the design of the vessel.
- The curve $2\beta^*/y$ is used to predict the commencement of secondary yielding during unloading. Note that during an unloading cycle, yielding occurs at a level of β equal to $2\beta^*$. Therefore, the minimum removal temperature gradient necessary to cause secondary yielding in the inside surface is $2\beta^*/y$.
- The curve β^{**}/y is the design curve. Given the temperature of the inside and outside surfaces of the vessel and hence the value of ΔT, one can calculate the value of $\beta^{**}=\alpha E\Delta T/(1-\nu)$, and by using this curve, one can obtain the value of m. As shown in this figure, the loading capacity of the vessel is significantly increased compared to the elastic design.

2.5. Thermal unloading

Depending on the geometrical factor m, and the amount of removed temperature gradient from the vessel, different deformation modes commence in the vessel.

At this point different deformation modes on the unloading phase are discussed when the vessel is under different temperature gradients, β^{**}.

Table 1
Values of c^{**}/a and d/a for several values of b/a

$m=b/a$	$m/m_1^{**}=c^{**}/a$	$m/m_2=d/a$
1.655	1.108	1.000
2.200	1.247	1.086
2.791	1.395	1.182
4.000	1.668	1.358
5.200	1.867	1.500
6.400	2.086	1.619

- At $m=1.655$, if the temperature gradient on loading is equal to β^{**}, then, during the unloading phase the second yielding initiates.
- For the case of $m<1.655$, since the relation $\beta^{**}<2\beta^{*}$ holds, if the level of loading temperature gradient is β^{**}, then the unloading condition is always elastic.
- Because the removing temperature gradient for the case of $m>1.655$ is always greater than $2\beta^{*}$, secondary yielding is always established in the vessel. In order to obtain the position of the second elasto-plastic yielding, d, the stresses in the elastic unloading phase could be calculated which means the following new yield criterion at this radius could be applied

$$(\sigma_\theta - \sigma_r)|_{r=d} = 2y \tag{25}$$

If the ratio b/d is designated by m_2, the relation between m_2 and β^{**}/y is as follows [9]:

$$\frac{\beta^{**}/y}{4(m-1)(m_2^2+m_2+1)}\left\{3m_2^3\left(1-\frac{4(m-1)\ln(m/m_2)}{(\beta^{**}/y)(m_2-1)}\right) - \frac{}{2}m_2(m_2^2+m_2+1)\right\} = 1 \tag{26}$$

In Table 1 for several values of b/a, the corresponding values for c/a and d/a are presented.

As said before, for the strengthening mechanism and also to increase the loading capacity of the vessel under a temperature gradient, the thermal autofrettage method can be used. To do this, after calculating the vessel dimensions (by considering known values for the loading temperature and the volume of the vessel) and before putting the vessel into the action, the vessel must experience a pre-working condition (loading–unloading cycle) under a temperature gradient of β^{**}/y.

3. The effect of convective heat transfer

In practice, only the temperatures of the internal and external fluids (usually air for the external fluid) in contact with the thick-walled vessel are specified. The temperature of the vessel surfaces cannot be obtained unless the effects of the thermal boundary layer on the inside and outside surfaces of the vessel are taken into consideration. Moreover, if the temperatures of the internal and external fluids are known, the relationship between ΔT (temperature gradient between the surfaces of the vessel) and ΔT_∞ (temperature gradient of the fluids) depends on the geometry of the vessel, which in turn is a design unknown parameter. Therefore, in order to reach an appropriate design condition, the relation between ΔT_∞, a, and b, must be derived.

Fig. 5(a) illustrates several different parameters of the vessel, useful for this analysis. For example $T_{\infty,\mathrm{in}}$, T_{air}, $h_{\infty,\mathrm{in}}$ and h_{air} are the temperatures of the internal and external fluids, and the convective heat transfer coefficients corresponding to the inside and outside surfaces of the vessel, respectively. In Fig. 5(b), the variation of temperature in regions where the boundary layers form and corresponding wall thickness of the vessel are shown. $R_{\mathrm{conv,in}}$, R_{cond}, and $R_{\mathrm{conv,out}}$ are the convective thermal resistance of the inside surface, conductive thermal resistance of the wall thickness, and convective thermal resistance of the outside surface of the vessel, respectively. q is the amount of heat flux, which under a steady-state condition becomes a constant value.

The values of any of these thermal resistances can be calculated as [10]

$$R_{\mathrm{cond}} = \frac{1}{4\pi K}(1/a - 1/b) \tag{27}$$

$$R_{\mathrm{conv,in}} = \frac{1}{4\pi h_{\infty,\mathrm{in}} a^2} \tag{28}$$

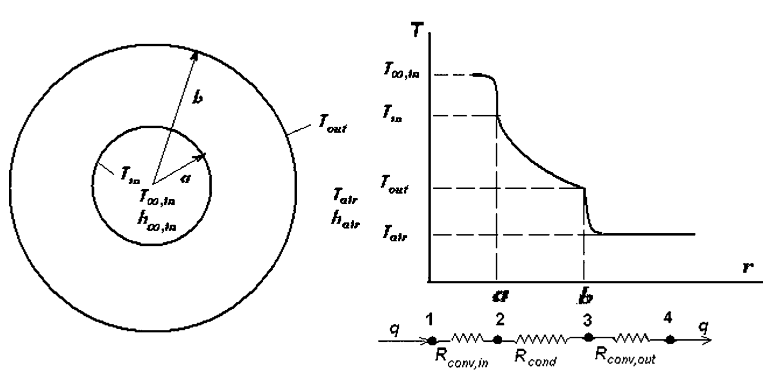

Fig. 5. (a) Definition of different parameters used in the design of the vessel. (b) Variation of temperature within and outside of the vessel.

$$R_{\text{conv,out}} = \frac{1}{4\pi h_{\text{air}} b^2} \tag{29}$$

Under steady-state heat transfer conditions, in Fig. 5(b) the value of q is the same between points 1–4 and 2–3, i.e. $(q=q_{23}=q_{14})$. By equating the values of these qs we have [10]

$$\Delta T = \frac{(1/K)[1/a - 1/b]}{1/h_{\text{in}}a^2 + (1/K)[1/a - 1/b] + 1/h_{\text{air}}b^2}\Delta T_\infty \tag{30}$$

in which K is the coefficient of heat conduction. By combining Eqs. (23) and (30) and considering $\beta^{**} = \alpha E \Delta T/(1-\nu)$, the relationship between ΔT_∞, a, and b is as follows [9]

$$\frac{\beta_\infty^{**}}{y}\frac{[1/a - 1/b]}{K/h_{\infty,\text{in}}a^2 + [1/a - 1/b] + K/h_{\text{air}}b^2} = \frac{m-1}{m_1^{**}-1}\frac{2[m_1^{**3}(3\ln(m/m_1^{**})+1)-1]}{m_1^{**}(2m_1^{**2} - m_1^{**} - 1)} \tag{31}$$

where $\beta_\infty^{**} = \alpha E \Delta T_\infty/(1-\nu)$. It should be noted that the coefficients of conductive heat transfer of the inside and outside surfaces of the vessel, are functions of many parameters such as the size of the surfaces, temperatures of fluids, etc.

3.1. Spherical vessel in contact with air

In this case, we consider only the effect of heat transfer from the outside surface of the vessel. The external fluid, which is in contact with the outside surface of the vessel, is air. Because in the design condition the volume of the vessel is prescribed, the inside radius a is also prescribed. In Eq. (31), h_{air} is a function of air properties, temperature of the outside surface of the vessel, and the geometry of the vessel. Hence, in order to design the vessel properly we need an explicit relation for this parameter. In [10], the following relation for h_{air} is given:

$$h_{\text{air}} = Nu_D \frac{k}{D} \tag{32}$$

in which, k is the conductive heat transfer coefficient of the air, D is the outside diameter of the vessel ($D=2b$), and Nu_D is the Nuselt Number which changes according to the following relation [10]

$$Nu_D = 2 + (0.4Re_D^{1/2} + 0.06Re_D^{2/3})Pr^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4} \tag{33}$$

where Reynolds Number, Re_D is defined as [10]

$$Re_D = \frac{V_{\text{air}}D}{\nu} \tag{34}$$

In the above equations, Pr, μ_∞, μ_s, V_{air}, and ν are the Prandtl number, air viscosity, air viscosity at the outside surface temperature, air velocity around the vessel, and the kinematic viscosity of the air, respectively. The parameters Pr, μ, ν, and k are all functions of the air temperature. In Table 2 the values of these parameters are given for different values of air temperature.

Table 2
Thermophysical properties of the air at atmosphere pressure [10]

$T°$ (K)	$\mu \times 10^7$ (N s/m^2)	$\nu \times 10^6$ (m^2/s)	$k \times 10^3$ (W/mk)	Pr
300	184.6	15.89	26.3	0.707
350	208.2	20.92	30.0	0.700
400	230.1	26.41	33.8	0.690
450	250.7	32.39	37.3	0.686
500	270.1	38.79	40.7	0.684

Table 3
Physical and mechanical properties of mild steel [4,10]

Modulus of elasticity, E (GPa)	Yield stress, y (MPa)	Poisson's ratio, ν	Heat expansion coefficient, α (1/K)	Convective heat transfer coeff., K (W/m K)
207	240	0.3	1.15e−5	13.4

Now, using the value of $h_{\infty,\text{in}}$ as ∞ in Eq. (31), and using the values of thermo-physical properties of air from Table 2, and also the values of physical and mechanical properties of *mild steel* from Table 3 and having the geometric dimensions, we can calculate the permissible temperature of the inside surface from the criterion of maximum plastic depth.

The variations of T_{in}^{**}, T_{in}^{***} versus b for the case of $a=1$ m, $T_{\text{air}}=300°$ K, $V_{\text{air}}=10$ m/s are shown in Fig. 6. T_{in}^{**} is the permissible temperature of the inside surface of the vessel, in which the outside surface of the vessel is held at $T_{\text{out}}=300°$ K. T_{in}^{***} is the permissible temperature of the inside surface imposed on the vessel, when the effect of heat transfer is considered.

Consider q_{23} and q, then by equating these values, we have

$$T_{\text{in}} = T_{\text{air}} + (R_{\text{conv,out}} + R_{\text{cond}})(1-\nu)\beta^{**}/E\alpha R_{\text{cond}} \tag{35}$$

Obviously the ratio $\beta^{**}/R_{\text{cond}}$ increases by increasing b and the total thermal resistance curve has an absolute minimum at

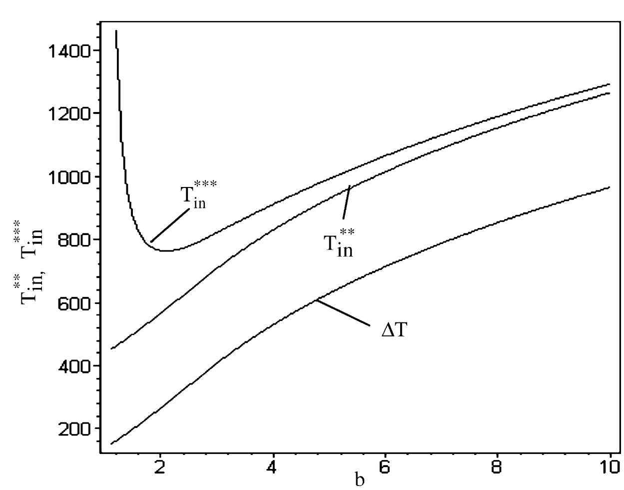

Fig. 6. Variations of permissible temperatures, versus b.

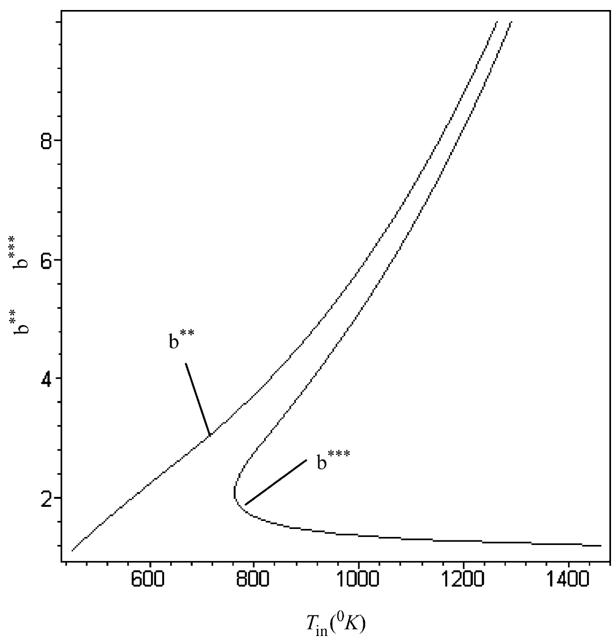

Fig. 7. Variations of optimum value of the outside radius, versus T_{in} (K).

$b=K/h_{air}$ [10]. Furthermore, we can see that the minimum value of T_{in}^{***} is 780° K and occurs for $b=2.2$. Therefore, for $T_{in}<780°$ K imposed on the vessel with an arbitrary radius, if the vessel yields then m_1 will be less than m_1^{**}. Suppose the internal radius and the temperature of the inside surface are known, we can get the optimum value of the outside radius of the vessel, using Eq. (31) and Tables 2 and 3.

The variations of b^{**} and b^{***} versus T_{in} for the case of $a=1$ m, $T_{air}=300°$ K, $V_{air}=10$ m/s are shown in Fig. 7; b^{**} is the optimum value of the outside radius of the vessel, in which the inside and outside surfaces of the vessel are held at the fixed temperatures T_{in}, and $T_{out}=300°$ K, respectively (Eq. (23)). Hence, b^{***} is the optimum outside radius of the vessel, when the effect of heat transfer is also considered.

As an interesting result of Fig. 7, in this case for $T_{in}<780°$ K we can consider any value for the external radius and for $T_{in}>780°$ K, we can find two optimum values for the outside radius of the vessel and also the difference between the two curves b^{**} and b^{***} is clear.

4. Conclusions

In this paper by considering a maximum plastic radius and using the concept of thermal autofrettage for the strengthening mechanism, the minimum thickness of a vessel for which the inside and outside surfaces of the vessel are held at fixed temperatures is obtained. In addition, because of the presence of convective heat transfer from the outside surface of the vessel, the optimum thickness of the vessel will be different from the vessel for which the inner and outer surfaces are held at fixed temperatures. This subject is also considered in this paper. This effect is presented as a new formula, but this formula has not a dimensionless form and could not be expressed as a function of b/a. For this reason, the design curves are only presented for several special cases. However, having on hand the surrounding conditions, the geometry of the vessel, and the material properties of the vessel, by using this formula problems of this nature can be solved.

Acknowledgements

The authors wish to express their gratitude to the Office of Research Affairs of Sharif University of Technology in providing partial financial aids to conduct this work.

References

[1] Boley BA, Weiner JH. Theory of thermal stresses. New York: Wiley; 1960.
[2] Nowaki W. Thermo-elasticity. Oxford: Pergamon Press; 1965.
[3] Timoshenko S, Goodier JN. Theory of elasticity. New York: McGraw-Hill; 1951.
[4] Johnson W, Mellor PB. Engineering plasticity. London: Ellis Horwood; 1983.
[5] Whalley E. The design of pressure vessels subjected to thermal stress-I, general theory for monoblock vessels. Can J Technol 1956;34(2): 268–75.
[6] Cowper JR. The eastoplastic thick-walled sphere subjected to a radial temperature gradient. Trans ASME 1960;27(3):496–500.
[7] Derrington MG, Johnson W. The onset of yield in a thick spherical shell subject to internal pressure and a uniform heat flow. Appl Sci Res Ser A 1958;7(4):408–14.
[8] Johnson W, Mellor PB. Elastic-plastic behaviour of thick-walled spheres of non-work-hardening material subject to a steady state radial temperature gradient. Int J Mech Sci 1961;4(1):147–54.
[9] Rezai Zarei A. Design of thick-walled vessels under high pressure and elevated temperature using elastoplastic analysis. MSc Thesis. Sharif University of Technology, Chapter 4; 1998.
[10] Incropera FP, Dewitt DP. Introduction to heat transfer. New York: Wiley; 1985.